CENTER VORTICES IN SU(2) GAUGE-HIGGS THEORY

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We show that center vortices and center dominance are present in some regions of the phase diagram of the SU(2) gauge-Higgs model, even though global center symmetry is broken, in this model, by the matter field.

1 Introduction

It is well known that "permanent" confinement $-$ in the sense of a non-zero asymptotic string tension $-$ is tied to the existence of a non-trivial, unbroken, global center symmetry of the underlying gauge theory. On the lattice, this global symmetry amounts to the invariance of the action upon multiplying all timelike links, at a fixed time, by the same center element. If this symmetry is unbroken, the vanishing of the Polyakov line VEV in an infinite spatial volume is guaranteed. If, instead, the global center symmetry is trivial, then the asymptotic string tension vanishes.[a] In the case of an unbroken Z_N center symmetry, the asymptotic string tension depends on the gauge group representation of the Wilson loops only through the N-ality of the represention; this means that confining vacuum fluctuations must somehow contrive to disorder only the center degrees of freedom of Wilson loop holonomies. These simple kinematical facts provide a strong motivation for the center vortex theory of confinement, which by now has abundant numerical support in SU(2) and (to a somewhat lesser extent) SU(3) lattice pure gauge theory.[1]

The addition of matter fields in the fundamental color representation breaks global center symmetry completely; there is no such symmetry in real QCD with its three generations of quarks. Confinement is then best described as "temporary", i.e. there is a linear potential for some finite range of quark separation, but at some point the static quark potential goes flat. It is not obvious that the center vortex picture, which is motivated by the N-ality properties of the non-vanishing asymptotic string tension, is relevant to the temporary confinement situation.

In order to address the relevance of center vortices in the absence of global center symmetry, three approaches come to mind. First, one could investigate a gauge theory with light fermions in the fundamental representation, such as real QCD. Here we would need to implement dynamical fermions on the lattice, which is numerically very challenging. A second approach is to study whether vortex degrees of freedom are somehow important in a gauge theory whose gauge group has a trivial center and trivial first homotopy group, the simplest case being the exceptional group G(2). This study is currently in progress,[b] and the results will be reported at a later time. The third (and by far the simplest) procedure is to add to the pure gauge theory a scalar field in the fundamental representation of the SU(2) gauge group, thereby breaking the global Z_2 center symmetry explicitly. It is

[a] We note that G(2) gauge theory is an example of, rather than an exception to, this general rule.
[b] In collaboration with K. Langfeld, H. Reinhardt, and T. Tok.

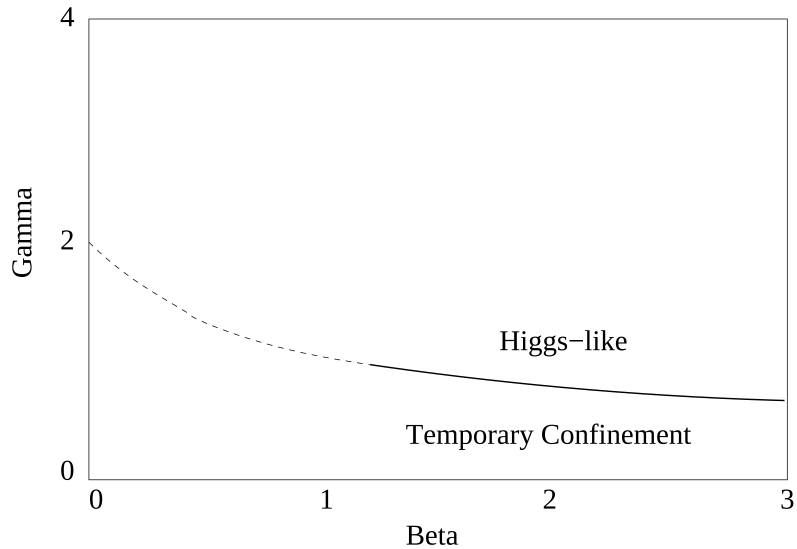

Figure 1: Schematic phase diagram of the SU(2) gauge-Higgs system. The solid line is a line of first-order phase transitions. The dashed line represents non-thermodynamic transitions in some non-local order parameter; the precise position of this line would depend on the order parameter chosen.

well known that there is a region of the SU(2) gauge-Higgs phase diagram in which temporary confinement holds, and there exists a linear potential up to some string-breaking distance. The rest of the phase diagram is Higgs-like, and there is no linear potential at any scale. We will concentrate on the temporary confinement region. The question is: Do vortex degrees of freedom account for the linear potential, where this potential exists? Are these degrees of freedom also sensitive to string breaking, and the leveling off of the static potential? In this contribution we will report the results of a very preliminary study of these issues.

2 SU(2) Gauge-Higgs Theory

In our numerical simulations, we use a version of SU(2) gauge-Higgs theory in which the scalar field has a fixed (or "frozen") modulus. For the SU(2) gauge group, the lattice action can be written in the form

$$S = \beta \sum_{plaq} \frac{1}{2}\mathrm{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{links} \frac{1}{2}\mathrm{Tr}[\phi^\dagger U \phi], \qquad (1)$$

where ϕ is SU(2) group-valued. This theory was first studied numerically long ago by Lang et al;[2] the phase diagram is sketched in Fig. 1. There is a line of first order transitions terminating at around $\beta = 1.2$. Above this line, the theory is Higgs-like, below the line, temporary confinement prevails. The Fradkin-Shenker–Osterwalder-Seiler theorem[3,4] assures us that the temporary confinement and Higgs-like regions are continuously connected; i.e. there are paths from one region to the other which do not encounter any thermodynamic singularity. Of course, there can be non-thermodynamic transitions in

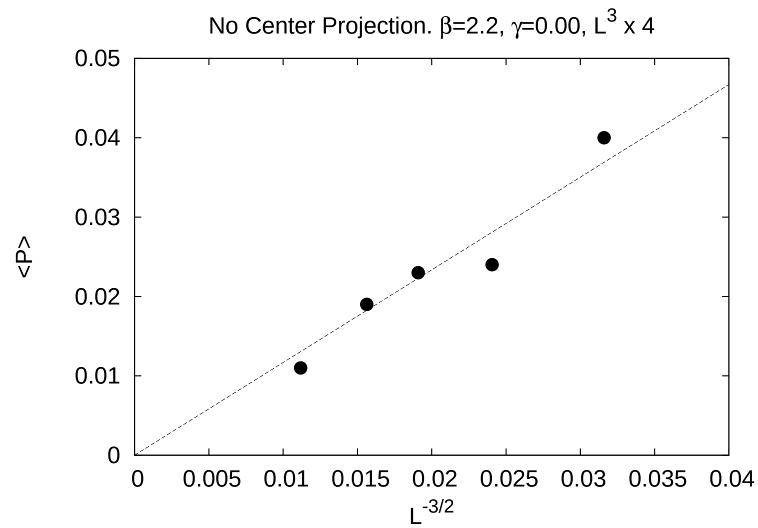

Figure 2: Polyakov line values, at $\gamma = 0.0$.

non-local observables. In the variable modulus version of gauge-Higgs theory, it was found that there is a line of center vortex percolation transitions, extending out from the line of first-order transitions and continuing down to $\beta = 0$.[5] This is known as a Kertész line. There are also symmetry-breaking transition lines. For example, in Coulomb gauge, there exists a remnant global symmetry, which is unbroken in the temporary confinement region, and broken in the Higgs-like region[6] (see also [7,8]). Again, this symmetry breaking coincides with the line of first-order transitions, and then extends to $\beta = 0$. But the remnant symmetry-breaking line and the percolation transition line do not coincide, beyond the line of first-order thermodynamic transitions,[9] and it seems likely in general that different non-local order parameters will have abrupt transitions along different lines in the phase plane, away from the first-order transition line.[c]

Our main interest is in the temporary confinement region, at a Higgs coupling γ where the effects of screening are visible. For this preliminary study, we have chosen to work mainly at $\beta = 2.2$ and $\gamma = 0.71$. The first order transition, at $\beta = 2.2$, occurs at about $\gamma = 0.9$.

We begin with a measurement of the Polyakov line at $\beta = 2.2$, on an $L^3 \times 4$ lattice. If $P(\mathbf{x})$ denotes the Polyakov line passing through the point $\{\mathbf{x}, t = 0\}$, then the quantity we measure is

$$\langle P \rangle \equiv \left\langle \frac{1}{L^3} \left| \sum_{\mathbf{x}} P(\mathbf{x}) \right| \right\rangle . \tag{2}$$

In the case of unbroken center symmetry, at $\gamma = 0$, we must find for $\beta = 2.2$ and $L_t = 4$ time extent that

$$\langle P \rangle \propto \sqrt{\frac{1}{L^3}}. \tag{3}$$

[c]In particular, it would be interesting to locate the transition line of the Pisa monopole operator[10] in this coupling plane.

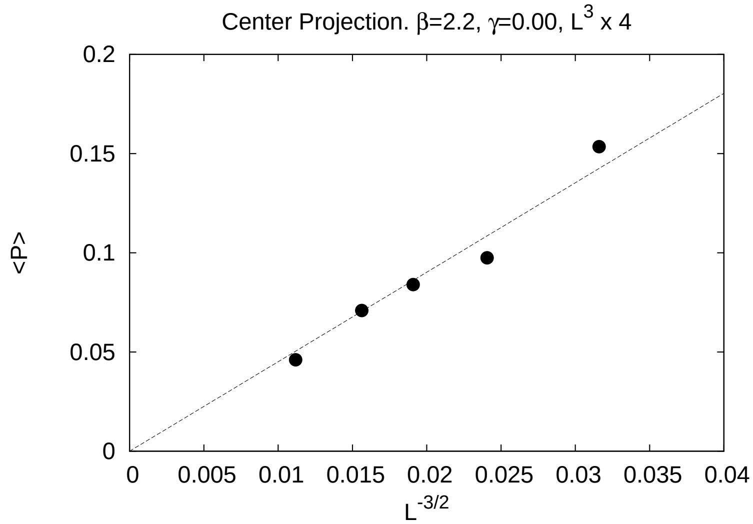

Figure 3: Center-projected Polyakov line values, at $\gamma = 0.0$.

The relevant data, for lattice sizes up to $20^3 \times 4$ is shown in Fig. 2. Fixing to direct maximal center gauge, and carrying out center projection[11] gives the result shown in Fig. 3. In both cases, the extrapolation to infinite volume is consistent with $\langle P \rangle = 0$.

By contrast, at $\beta = 2.2$ and $\gamma = 0.71$, we find that Polyakov lines on full and center-projected lattices do *not* seem to extrapolate to zero, as seen in Figs. 4 and 5. This establishes that at $\gamma = 0.71$ there is a small but detectable screening effect due to the dynamical scalar field, and this effect can be seen both with the full and center-projected lattice variables. This result is not really new; similar results for the variable modulus gauge-Higgs theory have been reported previously.[5]

Now let us look at the correlator of center-projected Polyakov lines $\langle P(x)P(x+R)\rangle$ at $\beta = 2.2,\ \gamma = 0.71$, on a $20^3 \times 4$ lattice. The data is shown in Fig. 6. The dashed line is a best fit to the data, for $R \geq 2$, by the function

$$f(R) = a + b\exp[-4\sigma R]. \tag{4}$$

From the fit we find $a = 0.0182,\ \sigma = 0.211$. Not surprisingly, a is quite close to the square of the VEV of the Polyakov line in center projection, which is $\langle P_{cp}\rangle = 0.135$ on the $20^3 \times 4$ lattice. In this way we see string-breaking, due to the dynamical matter field, on the center-projected lattice using Polyakov lines.

Having established some sort of “center dominance” for Polyakov lines, lets turn to spacelike Wilson loops. The first question to ask is whether P-vortices, identified via maximal center gauge fixing plus center projection, actually locate center vortices in unprojected SU(2) configurations in the temporary confinement region. Our standard test is to check whether $W_1/W_0 \to -1$ in the large-loop limit, where $W_n(C)$ represents a Wilson loop, computed from unprojected link variables, with the restriction that loop C is pierced by n P-vortices on the projected lattice This test seems to work out quite well for the gauge-Higgs system at $\beta = 2.2,\ \gamma = 0.71$, as seen in Fig. 7.

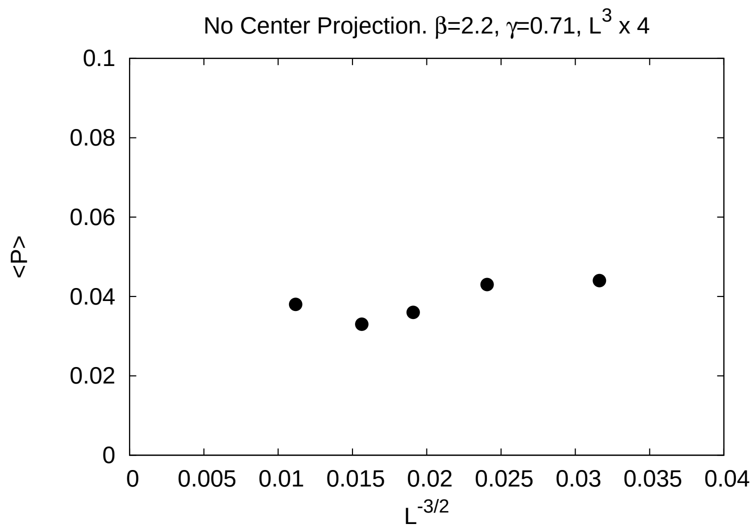

Figure 4: Polyakov line values, at $\gamma = 0.71$.

Finally we look at the Creutz ratios of spacelike Wilson loops, shown in Fig. 8. Notice that the center-projected Creutz ratios come out to ≈ 0.21, which is just what we found for σ extracted from the center-projected Polyakov line correlator. We also see that vortex removal sends the string tension to zero, just as in the pure gauge case.

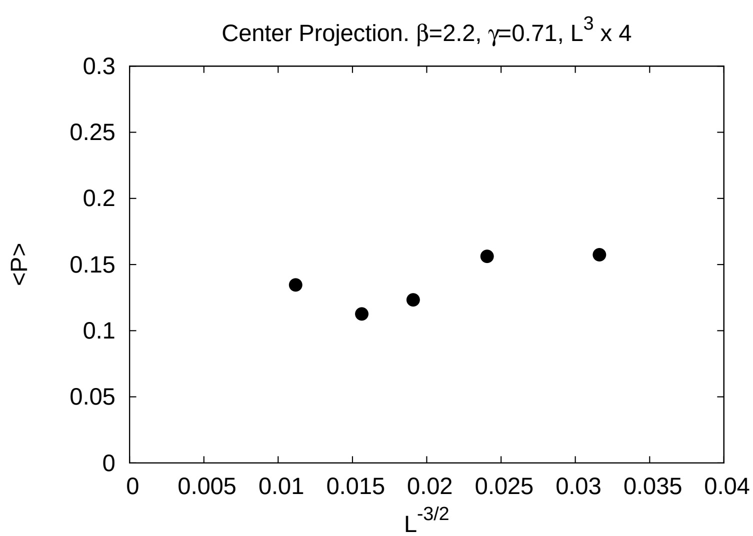

Figure 5: Center-projected Polyakov line values, at $\gamma = 0.71$.

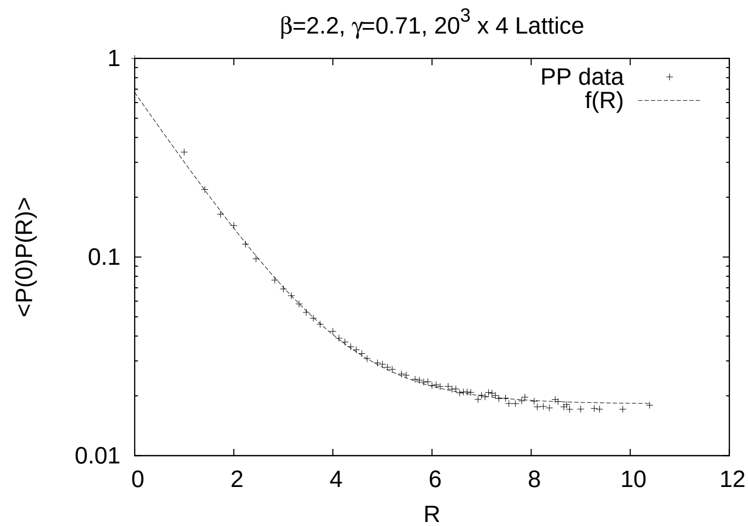

Figure 6: Polyakov line correlator $\langle P(0)P(R)\rangle$ on the center-projected lattice.

3 Conclusions

We have shown that in a coupling region where the scalar matter field has a small but measurable effect on Polyakov lines, center dominance is still a feature of the gauge-Higgs system. Vortex degrees of freedom are isolated, as in the pure gauge theory, via

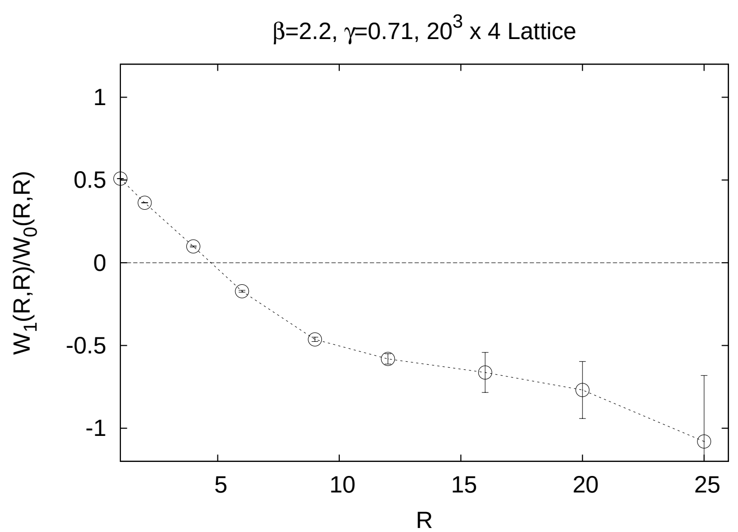

Figure 7: Ratio of “vortex-limited” Wilson loops. $W_1(C)$ is evaluated for loops pierced by a single P-vortex, $W_0(C)$ is evaluated for loops which are not pierced by any P-vortices.

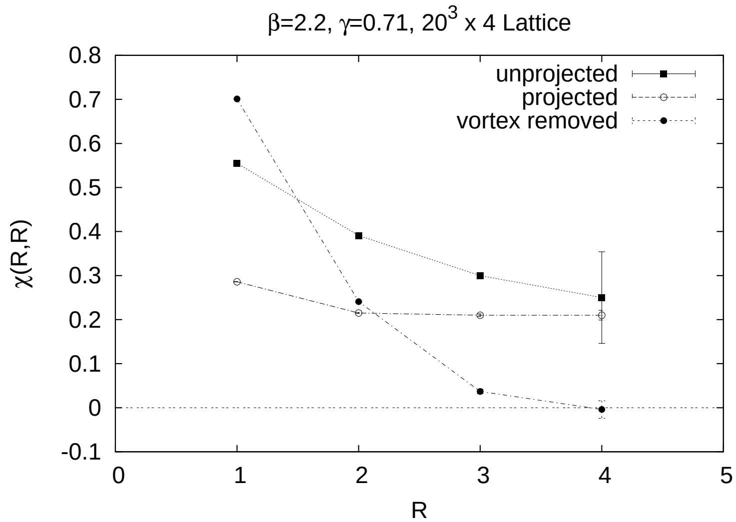

Figure 8: Creutz ratios in the gauge-Higgs theory for unprojected, center-projected, and vortex-removed configurations.

maximal center gauge fixing and center projection, and our standard tests on "vortex-limited" Wilson loops (W_1/W_0) show that vortices on the projected lattice locate thick center vortices in the unprojected configurations. We also find that the vortex degrees of freedom account for both the string tension and the string breaking found in Polyakov line correlators, while vortex removal removes the string tension (where it can be picked out of the noise). These results supplement previous findings in the variable modulus gauge-Higgs theory,[5] where it was shown that vortices on the projected lattice percolate in the temporary confinement region of the phase diagram, and do not percolate in the Higgs region.

The conclusion we draw from the SU(2) gauge-Higgs example is that the vortex confinement mechanism, and center dominance, can operate even when global center symmetry is broken by matter fields, and the asymptotic string tension is zero. This, of course, has important implications for the relevance of the vortex mechanism to real QCD. A related study of vortex degrees of freedom in G(2) lattice gauge theory is in progress, and will be reported at a later time.

Acknowledgments

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