Proceedings of the International School-seminar
"New Physics and Quantum Chromodynamics at external Conditions", pp. 1 – 4,
3-6 May 2009, Dnipropetrovsk, Ukraine

CENTER VORTICES AND DIRAC EIGENMODES

M. Faber[a], J. Greensite[b], U.M. Heller[c], R. Höllwieser[d], Š. Olejník[e],

[a,d]Atomic Institute, Vienna University of Technology, Wiedner Hauptstr. 8-10, A-1040 Vienna, Austria
[b]Physics and Astronomy Dept., San Francisco State University, San Francisco, CA 94132, USA
[c]American Physical Society, One Research Road, Box 9000, Ridge, NY 11961-9000, USA
[e]Institute of Physics, Slovak Academy of Sciences, SK–845 11 Bratislava, Slovakia

The center vortex model has been proposed as an explanation of confinement in non-Abelian gauge theories. Low-lying Dirac eigenmodes are related to the chiral properties of a gauge configuration. We investigate the correlation of center vortices and Dirac eigenmodes in SU(2) lattice gauge theory.

1 Introduction

Lattice QCD (LQCD) is the main tool for probing QCD in the non-perturbative regime, where QCD predicts quark confinement and chiral symmetry breaking. We work with lattices generated by Monte Carlo simulation of the tadpole improved Lüscher-Weisz pure-gauge action, mainly at coupling $\beta = 3.3$ (lattice spacing $a = 0.15$ fm) for the $SU(2)$ gauge group, which is very appropriate to study the mechanism and relation of these phenomena. Center-Projection is performed by Direct Maximal Center Gauge (adjoint Landau gauge), maximizing the squared trace of link variables $U_\mu(x)$ by the over-relaxation method. The mapping to link variables on the center-projected (or "vortex-only") lattice, for the $SU(2)$ gauge group, is given by $U_\mu(x) \to Z_\mu(x) = \text{signTr}[U_\mu(x)]$ and the link variables on vortex-removed lattices are defined as $U'_\mu(x) = Z_\mu(x)U_\mu(x)$.

2 Center vortex picture of confinement

Center vortices, quantized magnetic flux-lines, compress the gluonic flux into tubes and cause a linearly rising potential at large separations. This confinement mechanism was tested in many ways, for a detailed discussion see Ref. [1]. In Fig. 1 we show vortex limited Wilson loops W_n, with n vortices piercing the Wilson loop. As the loop area increases the Wilson loop approaches the limit $(-1)^n W_0$. Fig. 2 shows the effect of vortex removal on the Creutz ratio, the string tension vanishes whereas for the center-projected ("vortex only") configuration the Creutz ratio approaches the asymptotic string tension. Removing vortices also restores chiral symmetry [2], i.e. the chiral condensate vanishes, which is closely related to low-lying Dirac eigenmodes.

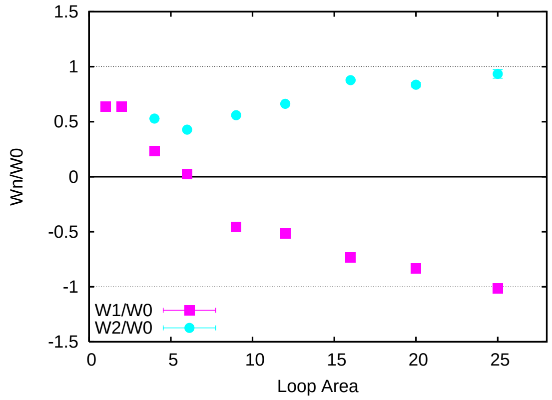

Figure 1. Ratios of vortex limited Wilson loops W_n on 20^4 lattice at $\beta = 3.1$.

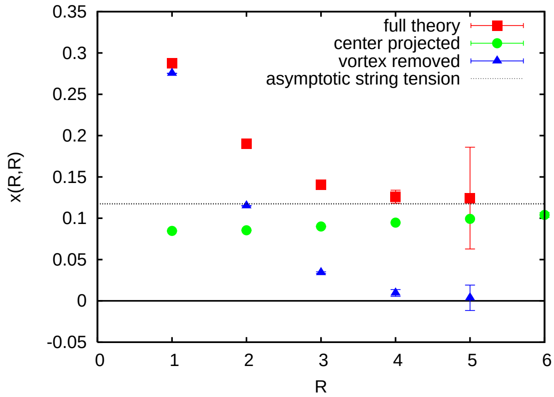

Figure 2. Creutz ratios at $\beta = 3.3$ for full, center-projected and vortex-removed 20^4 lattices.

e-mail: [a]faber@kph.tuwien.ac.at, [b]jgreensite@gmail.com, [c]heller@ridge.aps.org, [d]hroman@kph.tuwien.ac.at, [e]stefan.olejnik@gmail.com

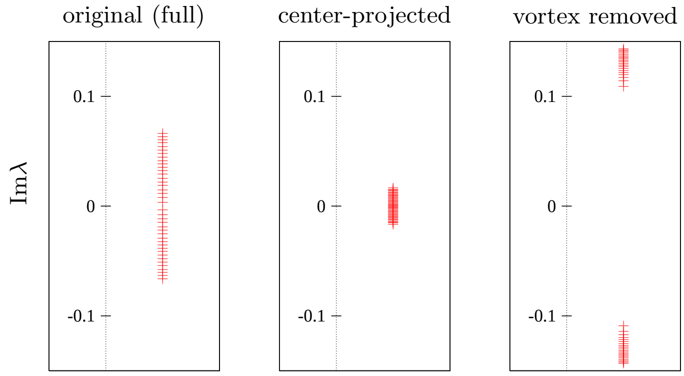

Figure 3. Dirac spectra of asqtad staggered fermions on a 20^4 lattice at $\beta = 3.3$ for full, center-projected and vortex-removed configurations.

3 Dirac eigenmodes and chiral symmetry breaking

The chiral condensate $\langle\overline{\psi}\psi\rangle$ is an order parameter for chiral symmetry breaking. According to the Banks-Casher relation [3] $\bar{\psi}\psi = \pi\rho(0)/V$, the chiral condensate is directly proportional to the density of near zero modes $\rho(0)$. Fig. 3 shows the first twenty eigenvalue pairs of the asqtad staggered Dirac operator [4] on a 20^4 lattice using antiperiodic boundary conditions at $\beta = 3.3$ for full, center-projected and vortex removed configurations. There is a clear gap in the vortex removed spectrum, indicating restored chiral symmetry, whereas in the vortex only case, the chiral condensate seems to be even enhanced in comparison to the original configuration. The vortex excitations of the center-projected lattice carry not only the information about confinement, but are also responsible for chiral symmetry breaking via the Banks-Casher relation. (see also [5, 6, 7])

4 Dirac eigenmode density and vortex correlations

Next we investigate the eigenmode density $\rho_\lambda(x) = \psi_\lambda^\dagger(x)\psi_\lambda(x)$, where $\psi_\lambda(x)$ is the normalized ($\sum_x \rho_\lambda(x) = 1$) eigenvector of the Dirac operator corresponding to the eigenvalue λ. We plot representative maximum density peaks for overlap and asqtad staggered eigenmodes on full configurations in Fig. 4 and for asqtad staggered eigenmodes on center-projected configurations in Fig. 5a. The overlap Dirac operator [8] does not provide reliable results on the singular gauge fields of center-projected configurations. In all other cases the figures show clearly sharp peaks in point-like regions of the lattice volume.

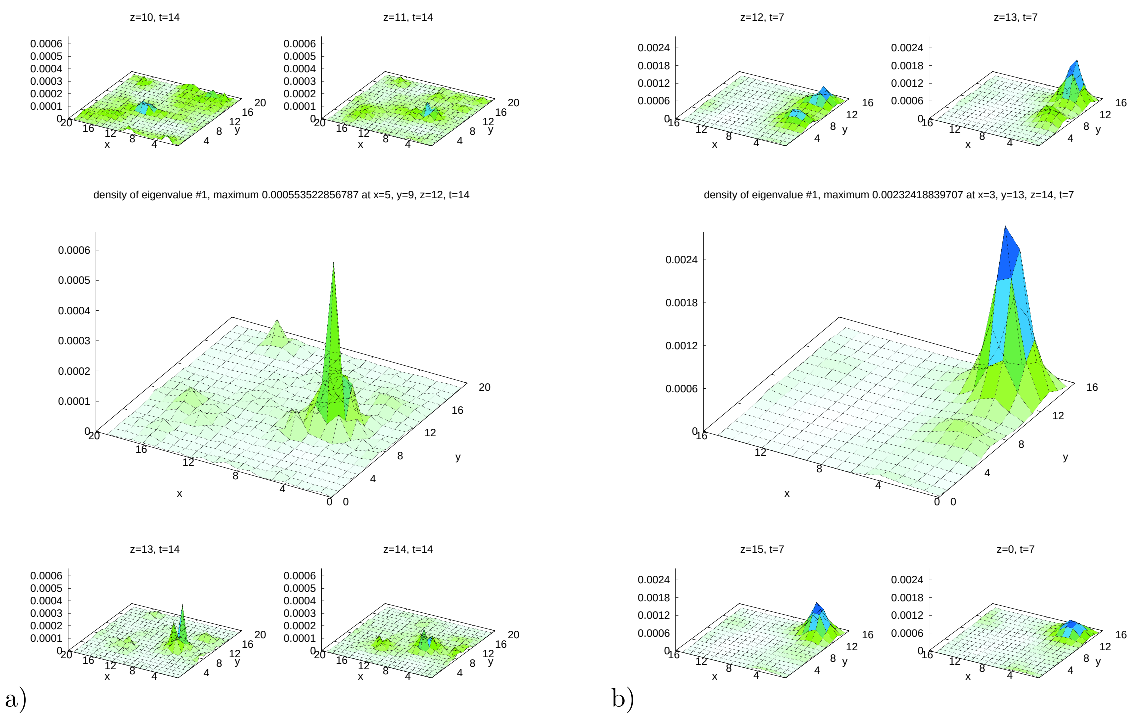

Figure 4. Maximum density peaks (center) of first a) asqtad staggered and b) overlap Dirac eigenmode on a full 20^4- resp. 16^4-lattice configuration, with upper (above) and lower (below) z-slices of the same t-slice.

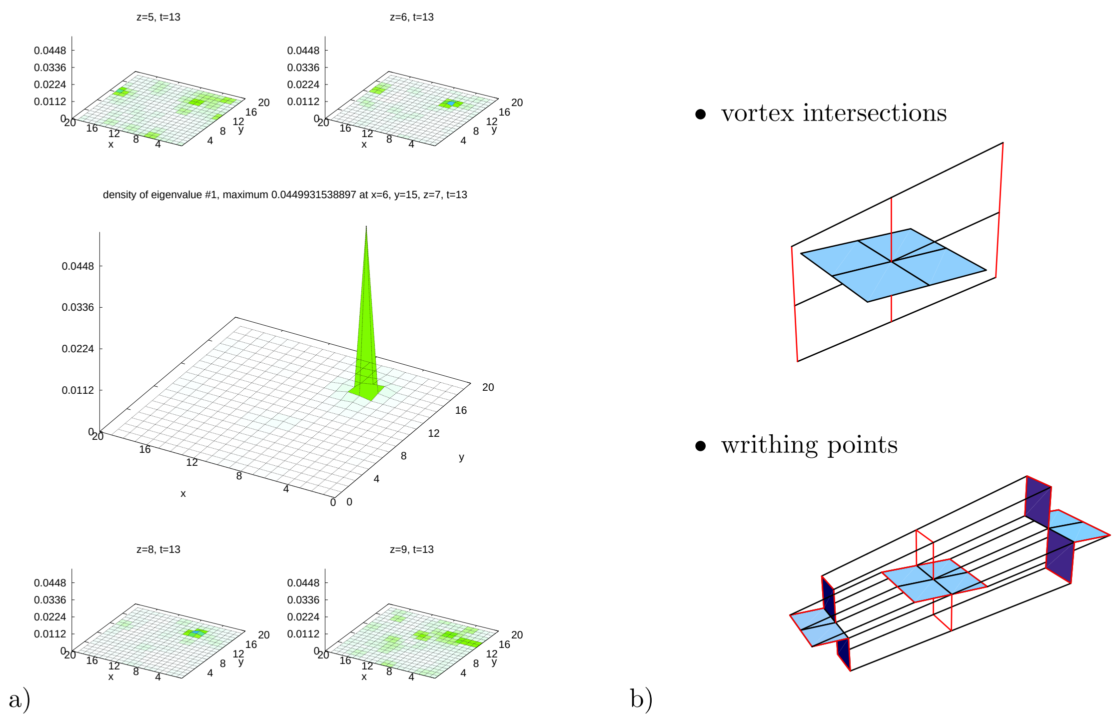

Figure 5. a) Maximum density peaks (center) of first asqtad staggered Dirac eigenmode on a center-projected 20^4-lattice configuration, with upper (above) and lower (below) z-slices of the same t-slice. b) Peaks correlate with topological charge sources, vortex intersection (above) and writhing points (below).

In Fig. 6 we present the vortex density in dependence of the distance (in lattice units a) from these eigenmode density peaks and find strong correlations between the eigenmode density and vortex structures. These dense vortex structures at the eigenmode density peaks are known to be sources of topological charge from the picture advocated by Engelhardt and Reinhardt [9], shown in Fig. 5b.

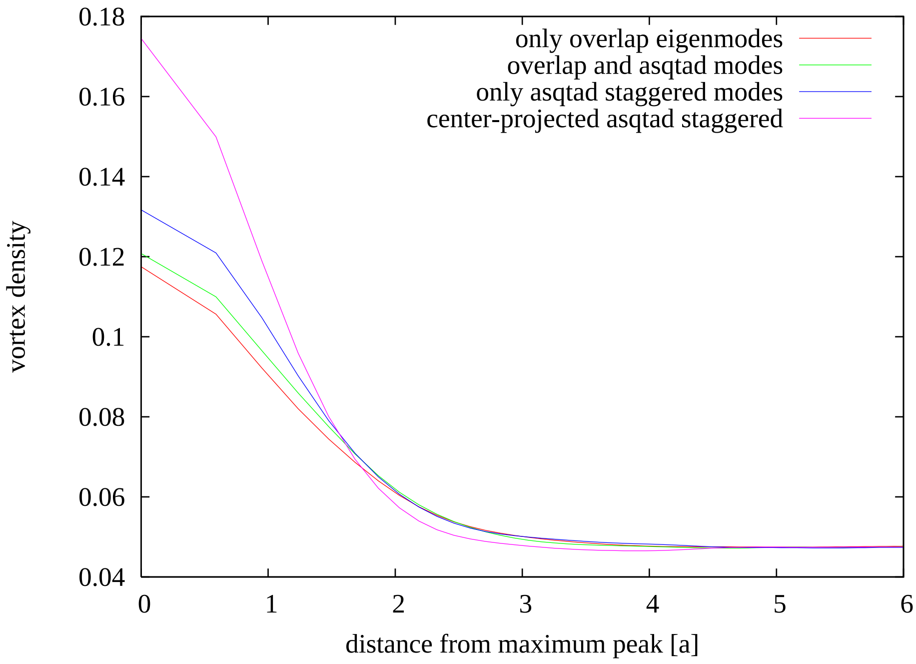

Figure 6. Vortex density in dependence of the distance from maximum Dirac eigenmode density peaks.

Finally, we directly correlate Dirac eigenmodes of different fermion operators by comparing the position of their eigenmode density peaks. In Fig. 7 we present a representative example of the eigenvalue spectra of overlap and asqtad staggered fermions on a full and the corresponding center-projected 16^4-lattice configuration.

We draw a line between two eigenvalues when their eigenmode density peaks match exactly. This is a rather interesting result since we find that the different eigenmode spectra do not provide the same physical contents at comparable energy levels, i.e. the correlations mix between different eigenvalues. The overlap zero mode for example appears as second center-projected and fourteenth full asqtad staggered mode.

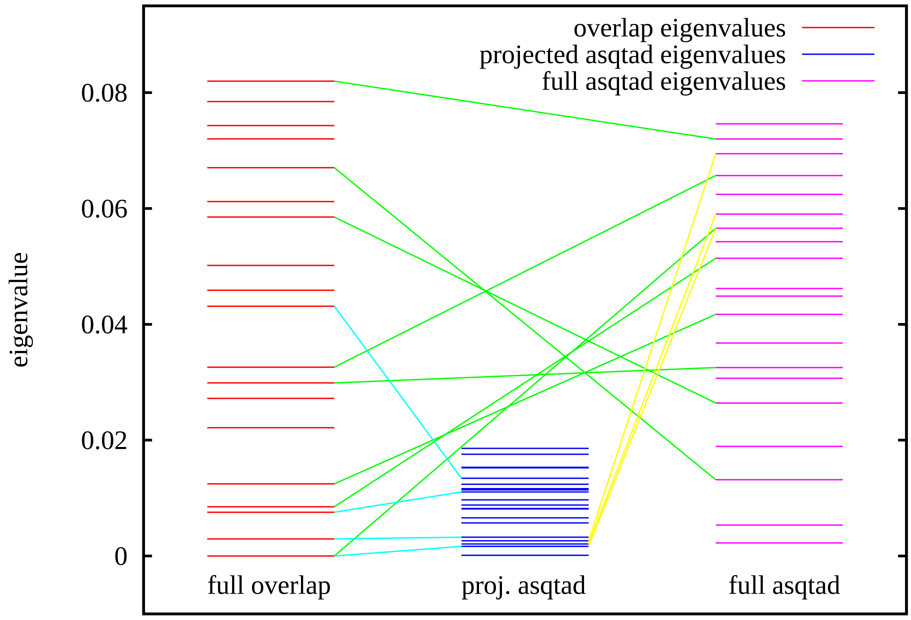

Figure 7. Dirac eigenvalue spectra on a 16^4-lattice configuration with eigenmode correlations (connection lines). For better presentation we just plot the absolute value of the overlap eigenvalues and not the Ginsparg-Wilson circle, which in fact makes no difference for the low-lying modes.

5 Conclusions

We presented significant results that the center vortex model not only explains confinement and topological charge, but also the breaking of chiral symmetry. We also find strong correlations between Dirac eigenmode, vortex and topological charge densities. These results were published in [10]. Nevertheless, the final result shows that different fermion realizations on the lattice may lead to slightly different results.

Acknowledgements. This research was supported by "Fonds zur Förderung der Wissenschaften" (FWF) under contract P20016-N16 (R.H.).

References

[1] J. Greensite, Prog. Part. Nucl. Phys. **51**, 1 (2003) [arXiv: hep-lat/0301023].
[2] P. de Forcrand and M. D'Elia, Phys. Rev. Lett. **82**, 4582 (1999) [arXiv:hep-lat/9901020].
[3] T. Banks and A. Casher, Nucl. Phys. **B169**, 103 (1980).
[4] Kostas Orginos, Doug Toussaint and R.L. Sugar, Phys. Rev. **D60**, 054503 (1999) [hep-lat/9903032]; G.P. Lepage, Phys. Rev. **D59**, 074502 (1999) [arXiv:hep-lat/9809157].
[5] C. Alexandrou, P. de Forcrand and M. D'Elia, Nucl. Phys. **A663**, 1031 (2000) [arXiv:hep-lat/9909005].
[6] F. V. Gubarev, S. M. Morozov, M. I. Polikarpov and V. I. Zakharov, [arXiv:hep-lat/0505016].
[7] V. G. Bornyakov, E. M. Ilgenfritz, B. V. Martemyanov, S. M. Morozov, M. Müller-Preussker and A. I. Veselov, Phys. Rev. **D77**, 074507 (2008) [arXiv:hep-lat/0708.3335].
[8] R. Narayanan and H. Neuberger, Nucl. Phys. **B443**, 305 (1995) [hep-th/9411108]; H. Neuberger, Phys. Lett. **B417**, 141 (1998) [hep-lat/9707022].
[9] M. Engelhardt, Nucl. Phys. **B585**, 614 (2000) [arXiv:hep-lat/0004013]; M. Engelhardt and H. Reinhardt, Nucl. Phys. **B567**, 249 (2000) [arXiv:hep-th/9907139].
[10] R. Höllwieser, M. Faber, J. Greensite, U.M. Heller and Š. Olejník, Phys. Rev. **D78**, 54508 (2008) [arXiv:hep-lat/0805.1846].